Modeling the Efficiency Equation of a Solar Box Cooker

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***Abstract*-The mundane task undertaken by household in search for source of energy (i.e. wood) for cooking, coupled with the glaring implications of defrost rations associated with it, this study deals with the modeling of the efficiency of a solar cooker. The authors reviewed previous work on same subject in different locations around the world. An initial study begins with a basic equation representing heat change by a body. The methodology employed involved the calculation of energy output from a solar cooker which was based on the change in temperature, mass and specific heat capacity of the water used. The paper however outline how people consistently involved themselves in cutting down trees all in attempt to look for energy for cooking food with less or no concern to consequences of their action in an attempt to reduce or completely eradicate the above action, a mathematical modeling has been developed which can be used by engineers to adjust certain parameters before the real construction began. The equation is therefore found to be useful in calculating the efficiency of any solar cooker to be constructed under the same condition.**

I. INTRODUCTION

Cooking is an important activity that occurs to prepare food for human survival. It is a well known fact that, most prominent cooking energy in most developing world is fuel-wood. It is a common practice in Africa, to see women and their children carrying heavy load of fire wood on their heads and backs over which they have to spend hours collecting miles from their homes (Metcalf, 2004). Garba & Atiku (1996) reported that, for more than a third of the world's people, the real energy crisis is thus a daily scramble to find wood they require to cook food. They further contended that, the demand for fuel-wood which is traditionally the most popular source of energy for domestic cooking is growing much more than the supply. In Nigeria like most of African countries, the dependence on fuel-wood is enormous (Garba et al Ibid), except where it is extremely scarce, and under such situation it is supplemented with other available and affordable energy options.

One of the alternatives to fuel-wood energy is solar energy, but the major problem with the use of solar energy is that, it is very dilute and fluctuates with time and weather condition. The economic feasibility of solar energy utilization depends upon efficient collection, conversion and storage. The efficient utilization of solar energy for heating, cooking and process application requires the use of flat plate or focusing collector system which first captures as much as possible, incoming radiation and then deliver a high fraction of the captured energy which is generally used as a working fluid.

The availability of solar energy depends on the time of the day, the climate and other meteorological parameters, such as clouds, rains and other atmospheric conditions. The time variation of solar radiation is generally out of phase with the demand, thus solar energy is viable only when thermal storage system is integrated. Therefore the aims and objectives of this study include determination of the appropriate mathematical modeling to enhance functioning of the system by varying parameters such as the collector, oven size and the nature of the transparent cover used for the system and whether the modeled equation of a given oven could be used by engineers in the future to construct a better solar cooker by estimating the efficiency of the solar cooker through computer simulation.

II. THEORETICAL FRAME WORK

Various studies were conducted in the past and different results were revealed. Below are some works conducted by some researchers at different locations around the world.

Nahar (2001) designed a double reflector hot box solar cooker with a transparent insulation material (TIM), and compared the performance with a single reflector solar cooker without TIM. The efficiencies were 30.5% and 24.5% for cooker with and without a TIM respectively during winter season at Jodhpur. The energy saving by use of a solar cooker with TIM has been estimated to be 1485 MJ of fuel equivalent per year. Sharaf (2002) revealed the concept of conical focus and explained the design of a solar cooker. The solar cooker was practically tested for grilling both white and red meat in a record time. Garba et al, (Ibid) developed a mathematical model for solar cooker. They obtained an energy balance equations for the different components of the cooker other than the plate as follows:

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Vessel and Its Contents;

$$\left(Mc_p\right)_v \frac{dT_v}{dt} = Q_b + Q_{c,a-v} + Q_{r,p-v} + Q_{r,g1-v} + Q_{s,p-v} + \left(Q_c + Q_r\right)_{v,c-v} \quad \rightarrow 2.1$$

Vessel Cover;

$$\left(M c_p\right)_{vc} \frac{d T_{vc}}{d t} = \alpha_{vc} S - Q_{c,vc-a} - Q_{r,vc-g1} - \left(Q_c + Q_r\right)_{vc-v} \quad \rightarrow 2.2$$

Glass Cover (First Layer);

$$\left(Mc_p\right)_{g1} \frac{dT_{g1}}{dt} = Q_{c,a-g1} + Q_{r,vc-g1} + Q_{r,\rho-g1} + Q_{r,s\rho-g1} S_o \left(1 - T_g - T_\rho\right)^2 - Q_{r,g1-v} - \left(Q_c + Q_r\right)_{g1-g2} \quad \rightarrow 2..3$$

Glass Cover (Second Layer);

$$\left(Mc_p\right)_{g2} \frac{dT_{g2}}{dt} = S_o \left(1 - T_g - \rho_g\right)^2 + \left(Q_c + Q_r\right)_{g1-g2} - Q_{c,g2} - amb - Q_{r,g2} - sky \quad \rightarrow 2.4$$

And Side Plates;

$$\left(M c_p\right)_{s\rho} \frac{d T_{s\rho}}{d t} = Q_{c,\rho-a} + Q_{c,vc-a} + Q_{c,s\rho-a} - Q_{c,a-c} - Q_{c,a-g1} \quad \rightarrow 2.5$$

III. METHODOLOGY

Solar thermal cooker basically depends on the solar collector used. In the energy analysis of the solar cooker, the energy gained by the solar cooker (that is cooking power) is considered as the energy output from the solar cooker (Huseyin, Serdar, & Ali 2004). The calculation of the energy output from a solar cooker is based on the change in water temperature ($T_{wf} - T_{wi}$), mass of the water M_w and the specific heat capacity of water, c_w as shown in the equation below;

$$E_o = M_w c_w \left(T_{wf} - T_{wi}\right) \quad \rightarrow 3.1$$

But, the time derivative of equation 3.1 yields,

$$P_o = \frac{M_w c_w \left(T_{wf} - T_{wi}\right)}{Dt} \quad \rightarrow 3.2$$

The useful energy from the collector is defined as the difference between solar flux absorbed by the absorber and heat losses from the absorber suggested by (Sharma,Bhosale,Kedere,&Nayak , 2006) as

$$Q = h_o AI - U\left(T_{abs} - T_{amb}\right)A \quad \rightarrow 3.3$$

Since the absorber temperature varies along absorber surface during operation, and it is difficult to define and measure absorber temperature, the mean fluid temperature T_m is used as suggested by Sharma (Ibid)

$$T_m = \frac{(T_{in} + T_{out})}{2} \quad \rightarrow 3.4$$

Where T_{in} and T_{out} are inlet and outlet temperature of fluid respectively.

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Therefore, eqn. 3.3 becomes

$$Q = h_o AI - U\left(T_m - T_{amb}\right) \quad \rightarrow 3.5$$

On the other hand, Sharma & Kazunobu, (2005) gave relation on the total heat gained by the working fluid (in this case water) can be obtained from the following relation

$$Q_w = mc_w \int_0^t \left(T_o - T_i\right) dt \quad \rightarrow 3.6$$

According to American Society of Association Executives (ASAE) (2003), the cooking power due to change in water temperature contained in a cooking vessel at 10 min. (600sec) can be written as

$$P_i = \frac{M_w c_w \left(T_2 - T_1\right)}{600\,\text{sec.}} \quad \rightarrow 3.7$$

Where

T_1 =initial water temperature

P_i = interval cooking power

T_2 = final water temperature

Standardizing cooking power required that cooking power for each interval should be corrected to a standard insolation of 700W/m^2 ASAE (Ibid), that is, multiplying the interval observed in cooking power by 700W/m^2 and dividing through by the average interval insolation recorded during the corresponding intervals as,

$$P_s = P_i \left(\frac{700}{I_i}\right) \quad \rightarrow 3.8$$

Where; P_s = standardized cooking power, and

I_i =average internal solar insolation

The basic equation describing an energy balance on the thermal mass within the cooking vessel as reported by Shawn (2001) is

$$M_w c_w \Delta T = h_o IAt \quad \rightarrow 3.9$$

Solving for time t from equation 3.9 yields

$$t = \frac{M_w c_w \Delta T}{h_o AI} \quad \rightarrow 3.10$$

The solar cooker is a heat storage collector without any power transported out of the system. The energy balance of this system with the equation for the collector efficiency as suggested by Duffie and Beckham, (1991) is

$$M_w c_w \frac{dq}{dt} = \left\{h_o I - kq\right\} A \quad \rightarrow 3.11$$

Equation 3.11 can be solve as follows

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$$\frac{M_w c_w dq}{(h_o I - kq)} = Adt \quad \rightarrow 3.12$$

Or

$$\frac{dq}{(h_o I - kq)} = \frac{A}{M_w c_w} dt \quad \rightarrow 3.13$$

Integrating both sides, gives

$$-\ln(h_o I - kq) = \frac{At}{M_w c_w} + \ln a \quad \rightarrow 3.14$$

Solving further yields

$$\ln(h_o I - kq) + \ln a = -\frac{At}{M_w c_w} \quad \rightarrow 3.15$$

Also,

$$\ln a(h_o I - kq) = -\frac{At}{M_w c_w} \quad \rightarrow 3.16$$

Therefore

$$a(h_o I - kq) = e^{-\frac{At}{M_w c_w}} \quad \rightarrow 3.17$$

Solving further to have

$$q = \frac{1}{k}\left[h_o - \frac{e^{\frac{At}{M_w c_w}}}{a}\right] = \frac{1}{k}\left[h_o - a e^{\frac{At}{M_w c_w}}\right] \quad \rightarrow 3.18$$

For simplicity, two constants are defined; they are for the flat-plate collectors with the global solar radiation I_{glb} as advanced by Hafner, Faber & Schwarzer (1999) these are

$$K_1 = \frac{I_{glb} h_o A}{M_w c_w} \quad \text{and} \quad \rightarrow 3.19$$

$$K_2 = \frac{kA}{M_w c_w} \quad \rightarrow 3.20$$

The collector's performance can be found using Sharma et al, (2005) fraction given by

$$f = \frac{Q_{Wstored}}{G} \quad \rightarrow 3.21$$

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The value of f depends on the load used in the cooker, and G can be calculated using

$$G = t\,a\,A\int_0^1 I_t dt \qquad \rightarrow 3.22$$

Moreover, from equations 3.19 and 3.20, we can get an amount of heat q the cooker needs to heat an amount of water at a time t as

$$\theta = \frac{K_1}{K_2} - \frac{K_1}{K_2} e^{-K_2 t} \qquad \rightarrow 3.23$$

And

$$\theta - \frac{K_1}{K_2} = \frac{K_1}{K_2} e^{-K_2 t} \qquad \rightarrow 3.24$$

$$\left(q - \frac{K_1}{K_2}\right) X \frac{K_2}{K_1} = e^{-K_2 t} \qquad \rightarrow 3.25$$

Solving further, equation 3.24 becomes

$$t = -\frac{1}{K_2} \ln\left(1 - \frac{K_2}{K_1} q\right) \qquad \rightarrow 3.26$$

IV. MODEL DESIGN OF THE EFFICIENCY EQUATION

Equation 3.1 represents the heat gained by the working fluid (water) without losses. If we take losses into account, we now have a new equation as

$$\frac{M_w c_w \left(T_f - T_i\right)}{\Delta t} = Q_{Loss} \qquad \rightarrow 4.1$$

Where Q_{Loss} is the losses the cooker suffered from and $Q_{Loss} = uA\Delta T$

So, equation 4.1 will becomes

$$\frac{M_w c_w \left(T_f - T_i\right)}{\Delta t} = uA_c \Delta T \qquad \rightarrow 4.2$$

Solving further yields

$$M_w c_w \left(T_f - T_i\right) = uA\left(T - T_a\right)\Delta t$$

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$$\left(T_f - T_i\right) = \frac{uA\left(T - T_a\right)\Delta t}{M_w c_w}, \text{ and}$$

$$T_f = \frac{uA\left(T - T_a\right)\Delta t}{M_w c_w} + T_i \qquad \rightarrow 4.3$$

Equation (4.3) above, is an equation for finding the final temperature of water. Equating equations 3.3 and 3.7 above, to have

$$\frac{M_w c_w \left(T_{wf} - T_{wi}\right)}{t} = h\,AI - u\left(T_{abs} - T_{amb}\right)A \qquad \rightarrow 4.4$$

Multiplying both sides of the above equation by t to have

$$M_w c_w \left(T_{wf} - T_{wi}\right) = t\left(h\,AI - uA\left(T - T_a\right)\right) \qquad \rightarrow 4.5$$

And

$$M_w c_w \left(T_{wf} - T_{wi}\right) = h\,AIt - uAt\left(T - T_a\right) \qquad \rightarrow 4.6$$

So,

$$M_w c_w \left(T_{wf} - T_{wi}\right) + uAt\left(T - T_a\right) = h\,AIt \qquad \rightarrow 4.7$$

Making h the subject of the above relation yields

$$h = \frac{M_w c_w \left(T_{wf} - T_{wi}\right) + uAt\left(T - T_a\right)}{AIt} \qquad \rightarrow 4.8$$

Hence,

$$h = \frac{M_w c_w \left(T_{wf} - T_{wi}\right)}{AIt} + \frac{u\left(T - T_a\right)}{I} \qquad \rightarrow 4.9$$

Since liquid does not have definite shape, and needs to be put in a container, the heat capacity of the container which is aluminum must be considered. So equation 4.9 becomes,

$$h = \frac{M_w c_w + M_{al} c_{al} \left(T_{wf} - T_{wi}\right)}{AIt} + \frac{u\left(T - T_a\right)}{I} \qquad \rightarrow 4.10$$

Equation 4.10 is the model equation for the efficiency of the solar box cooker.

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V. CONCLUSION

This paper has been able to remind the readers on the importance of cooking to people's lives. The paper further described the difficulties encountered by our people in the search of energy for cooking. It however outlined how people consistently involved themselves in cutting down trees all in an attempt to look for energy for cooking food with less or no concern to the consequences of their action. A simple solution has been suggested through the modeling of efficiency equation which has been developed through various heat gained equations used by several researchers in the past. The developed equation gives an opportunity for the engineers to determine the efficiency of the cooker before it is constructed.

Nomenclature

M =mass
Q = heat transfer rate
r = radius
S = solar irradiance on plate or vessel cover
S_0 = solar irradiance incident on glass cover 2
T = temperature
c_p = specific heat capacity

Subscripts

amb =ambient
a = air
c = convective or collector
g = glass cover
I = insulation
l = loss
p = plate
r = reflector
rc =reflective collector
S = sky, vessel side
v = vessel
vc = vessel cover

Greek letters

s = thickness
r = reflectivity
t = transmissivity
Q = Heat gain (in Watts)
h_o =Optical efficiency of the collector
A = Aperture area
I = Solar radiation
U = Overall heat transfer coefficient
T_{abs}= Absorber temperature
T_{amb} =Ambient temperature
m = flow rate of the water
c_w =specific heat capacity of water
T_o = outlet water temperature
T_i = inlet water temperature
$d\theta$ = temperature difference between the pot content and ambient
k = thermal loss coefficient
A =collector aperture area

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