Dynamics of Decision Making in Traditional Companies Using Three-Level Quadratic Programming Problem with Random Rough Coefficient in Constraints

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Abstract — This paper presents three-level quadratic programming problem with random rough coefficient in constrains. At the first phase of the solution algorithm, and to avoid the complexity of this problem, we begin with converting the rough nature in constraints into equivalent crisp form. At the second phase, a membership function is constructed to develop a fuzzy model for obtaining the optimal solution of the three-level quadratic programming problem. An auxiliary problem is discussed as well as an example is presented.

Index Terms—Rough parameter, Quadratic programming, three-level programming, rough in constraints

I. Introduction

Traditional companies used to be authoritarian and built around hierarchical organizational charts. The hierarchical model depends on levels, the decision makers (DMs) are in these levels, each level rules the next ones, and each DM wants to achieve his\here goals, that's by maximizing or minimizing his\her own objective function(s). This is the idea of multi-level programming, many researches deal with this point [].

In [13], Osman et. al. used the concepts of tolerance membership function and multi-level optimization at each level to develop a fuzzy decision model for generating satisfactory solution. In [3], Biswas used fuzzy goal programming procedure to solve multi-level quadratic programming problems, where all the parameters involved in objectives of each level and in system constraints are fuzzy defined. Also in [5], Emam solved bi-level integer multi-objective fractional programming problem using the cutting-plane algorithm.

Many real-world issues can be formulated as constrained optimization problems. In some situations in realistic decision making, the decision must be taken based on random constraints. The randomness of data is represented by rough parameters [14]. Rough set theory is another approach to vagueness, it has been proven to be an excellent mathematical tool dealing with vague description of objects [1,4,8,10,15].

In [7], Lin solved constrained optimization problems using genetic algorithm with the rough set theory, that is called the rough penalty genetic algorithm (RPGA), with the aim to effectively achieve robust solutions and resolving constrained optimization problems. In [9], Jana handled fuzzy rough multi-item economic production quantity (EPQ) model and developed constant demand. Infinite production rate have flexibility and reliability consideration in production process and demand dependent unit production cost and shortages under the restrictions on storage area, and by geometric programming (GP) technique solved the problem.

In [19], Xu turned from random rough variables into crisp equivalent model and introduced interactive method to obtain decision maker satisfying solution, applying a random rough simulation technique which can deal with random rough objective functions and constrains, combining with the genetic algorithm. Osman et. al. presented characterization of basic notions in parametric rough convex programming when parameters in constraints and roughness exist in the objective function [17] or when parameters in objective function and roughness in the feasible region [18].

In [6], Hamzehee constructed two LP problems with interval coefficients, one of them all its coefficients are upper approximations of rough intervals and the other all its coefficients are lower approximation of rough intervals. In [2], Alolyan solved linear programming problems with fuzzy parameters in the objective function and the constraints based on preference relations between intervals were investigated.

This paper is organized as follows: we start from Section 2, formulating the model of three-level quadratic programming problem under random rough coefficient in the constraints along with solution concept. In Section 3, the transformation of random rough coefficient into crisp is presented. In Section 4, fuzzy approach of an equivalent crisp three-level quadratic programming problem is suggested. In section 5, an example is provided to illustrate the developed results. Finally, section 6, some open points are stated for future research work in the field of rough multi-level quadratic optimization.

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II. Problem Formulation and Solution Concept

The three-level quadratic programming problem with random rough coefficient in the constraints (TLQPRP) may be written as:

[1st level]

$$\max_{x_1} f_1(x) = kx + \frac{1}{2} x^T C x, \quad (1)$$

Where x_2, x_3 solves

[2nd level]

$$\max_{x_2} f_2(x) = kx + \frac{1}{2} x^T C x, \quad (2)$$

Where x_3 solves

[3rd level]

$$\max_{x_3} f_3(x) = kx + \frac{1}{2} x^T C x, \quad (3)$$

subject to

$$G = \{x | Ax \leq l, x \geq 0\}. \quad (4)$$

Where f_1, f_2 and f_3 are the objective functions of the first level decision maker (FLDM), second level decision maker (SLDM), and third level decision maker (TLDM), C is $n \times n$ real matrix, and k is $(1 \times n)$ matrix, A is $(m \times n)$ real matrix contains random rough coefficients, and b is $(m \times 1)$ matrix, the vector of decision variables x is $(n \times 1)$ matrix partitioned between the three planners.

The first-level decision maker has control over the vector $x_1 \in R^{n_1}$, the second-level decision maker has control over the vector $x_2 \in R^{n_2}$ and the third level decision maker has control over the vector $x_3 \in R^{n_3}$, where $n = n_1 + n_2 + n_3$.

Definition 1

For any x_1 given by FLDM and x_2 given by SLDM, if decision-making variable x_3 is the optimal solution of TLDM, then (x_1, x_2, x_3) is a feasible solution of (TLQPRP).

Definition 2

If (x_1^0, x_2^0, x_3^0) is a feasible solution of the problem, no other feasible solution $(x_1, x_2, x_3) \in G$ exists, such that $f_i(x_1^0, x_2^0, x_3^0) \leq f_i(x_1, x_2, x_3)$ is the optimal solution for the TLQPRP.

Definition 3 [19]

Let Λ be a nonempty set, $\mathbf{C}$ be an σ-algebra of subset of Λ, Δ be an element in $\mathbf{C}$, and π be a nonnegative, real-valued, additive set function. Then $(\Lambda, \Delta, \mathbf{C}, \pi)$ is called a rough space.

Definition 4 [19]

Let v be a rough vector on the rough space $(\Lambda, \Delta, \mathbf{C}, \pi)$, and $f_j: R_n \to R$ be continuous functions, $j = 1, 2, \ldots, m$, then the upper trust of the rough event characterized by $f_j(v) \leq 0, j = 1, 2, \ldots, m$, is defined by

$$\mathrm{Tr}^*\{f_j(v) \leq 0, j = 1, 2, \ldots, m\} = \frac{\pi\{\lambda \in \Delta | f_j(v) \leq 0, j = 1, 2, \ldots, m\}}{\pi\{\Delta\}}$$

.

Definition 5 [19]

Let v be a rough vector on the rough space $(\Lambda, \Delta, \mathbf{C}, \pi)$, and $f_j: R_n \to R$ be continuous functions, $j = 1, 2, \ldots, m$. Then the lower trust of the rough event characterized by $f_j(v) \leq 0, j = 1, 2, \ldots, m$, is defined by

$$\mathrm{Tr}\{f_j(v) \leq 0, j = 1, 2, \ldots, m\} = \frac{\pi\{\lambda \in \Delta | f_j(v) < 0, j = 1, 2, \ldots, m\}}{\pi\{\Delta\}}$$

.

III. The Transformation of Rough Coefficient [19]

To convert the three-level quadratic programming problem with random rough coefficient in the constraints into the respective crisp equivalents for solving a trust probability constrains, we use the transformation of random rough coefficient. This process is usually hard work for many cases but the transformation process is introduced in the following theorem.

Theorem 3-1. [19]

Assume that random rough variable $\bar{c}_{ij}$ is characterized by $\bar{c}_{ij}(\lambda) \sim N(c_{ij}(\lambda), V_{ic})$ where:

$c_{ij}(\lambda)[(c_{ij}(\lambda)_{nx1} = c_{i1}(\lambda), c_{i2}(\lambda), \ldots, c_{in}(\lambda)^T)]$ is a rough variable and V_i^c is a positive definite covariance matrix. It follows that $c_i(\lambda)^T x = ([a, b], [c, d])$, (where $c \leq a \leq b \leq d$) is a rough variable and characterized by the following trust measure function:

$$\mathrm{Tr}\{c_i(\lambda)^T x \geq t\} = \begin{cases} 0 & \text{if } d \leq t, \\ \frac{d-t}{2(d-c)} & \text{if } b \leq t \leq d, \\ \frac{1}{2}\left(\frac{d-t}{d-c} + \frac{b-t}{b-a}\right) & \text{if } a \leq t \leq b, \\ \frac{1}{2}\left(\frac{d-t}{d-c} + 1\right) & \text{if } c \leq t \leq a, \\ 1 & \text{if } t \leq c. \end{cases}$$

Then, we have $\mathrm{Tr}\{\lambda \mid \Pr\{c_i(\lambda)^T x \geq f_i(x)\} \geq \delta_i\} \geq \gamma_i$, if and only if

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$$\begin{cases} b+R \le f_i \le d-2\gamma_i(d-c)+R & if\ c \le M \le d, \\ a+R \le f_i \le \frac{d(b-a)+b-2\gamma_i(d-c)(b-a)}{d-c+b-a}+R & if\ a \le M \le b, \\ c+R \le f_i \le d-(d-c)(2\gamma_i-1)+R & if\ c \le M \le a, \\ f_i \le c+R & if\ M \le c. \end{cases}$$

Where $M = f_i - \Phi^{-1}(1-\delta_i)\sqrt{x^T V_i^c x}$ and $R = \Phi^{-1}(1-\delta_i)\sqrt{x^T V_i^c x}$ and Φ is the standardized normal distribution and $\delta_i, \gamma_i \in [0,1]$ are predetermined confidence levels

Theorem 3-2. [19]

Suppose that random rough variables $\bar{e}_{rj}(\lambda)$ and $\bar{z}_r(\lambda)$ are characterized by $\bar{e}_{rj}(\lambda) \sim N(e_{rj}(\lambda), V_r^e)$, $\bar{z}_r(\lambda) \sim N(z_r(\lambda), (\sigma_r^e)^2)$ where $e_{rj}(\lambda), z_r(\lambda)$ are rough variables, and $V_r^e, (\sigma_r^e)^2$ are positive definition covariance. By Theorem3-1, we have that $e_{rj}(\lambda)^T x, z_r(\lambda)$ are rough variables; then $e_{rj}(\lambda)^T x - z_r(\lambda) = [(a,b),(c,d)](c \le a \le b \le d)$ is also rough variable. We assume that it is characterized by the following trust measure function:

$$\mathrm{Tr}\{e_r(\lambda)^T x - z_r(\lambda) \le t\} = \begin{cases} 0 & if\ d \le t, \\ \frac{t-c}{2(d-c)} & if\ c \le t \le a, \\ \frac{1}{2}\left(\frac{t-c}{d-c}+\frac{t-a}{b-a}\right) & if\ a \le t \le b, \\ \frac{1}{2}\left(\frac{t-t}{d-t}+1\right) & if\ b \le t \le d, \\ 1 & if\ d \le t. \end{cases}$$

Where $(e_{rj}(\lambda)_{mx1} = e_{r1}(\lambda), e_{r2}(\lambda), \ldots, e_{rm}(\lambda))^T$ Then, we have that $\mathrm{Tr}\{\lambda | \Pr\{\bar{e}_r(\lambda)^T x \le z_r(\lambda)\} \ge \theta_r\} \ge \eta_r$, if and only if

$$\begin{cases} c+2(d-c)\eta_r \le M \le a & if\ c \le M \le a, \\ \frac{2\eta_r(d-a)(b-a)+c(b-a)+a(d-c)}{d-c+b-a} \le M \le b & if\ a \le M \le b, \\ (2\eta_r-1)(d-c)+c \le M \le d & if\ b \le M \le d, \\ d \le M & if\ M \ge d. \end{cases}$$

Definition 6.

Assume that random rough variables $\bar{e}_{rj}(\lambda)$ and $\bar{z}_r(\lambda)$ are characterized by $\bar{e}_{rj}(\lambda) \sim N(e_{rj}(\lambda), V_r^e)$, $\bar{z}_r(\lambda) \sim N(z_r(\lambda), (\sigma_r^e)^2)$ where: $e_{rj}(\lambda), z_r(\lambda)$ are rough variables, $\mathrm{Tr}\{e_{rj}(\lambda)^T x \ge t, z_r(\lambda)^T x \ge t\} = \omega_{i1}, \ldots, \omega_{in}$ so ω is the minimum of $(\omega_{i1}, \ldots, \omega_{in})$.

Definition 7.

If x^* is an optimal solution of the three level quadratic programming problem under random rough coefficient in the constraints; no other feasible solution $x \in G$ exists, such that $f_1(x^*) \le f_1(x)$ so x^* is the optimal solution of the three level quadratic programming problem under random rough coefficient in the constraints with trust value ω.

IV. Fuzzy Approach of Three-Level Quadratic Programming Problem [13]

To solve an equivalent problem of the three-level quadratic programming problem with rough parameter in constraints, one first gets the satisfactory solution that is acceptable to FLDM, and then give the FLDM decision variable and goal with some leeway to the SLDM for him/her to seek the optimal solution, and to arrive at the solution which is closest to the optimal solution of the FLDM. This due to, the SLDM who should not only optimize his/her objective function but also try to satisfy the FLDM's goal and preference as much as possible. Then gets the satisfactory solution that is acceptable to SLDM, and then give the SLDM decision variable and goal with some leeway to the TLDM for him/her to seek the optimal solution, and to arrive at the solution which is closest to the optimal solution of the SLDM. This due to, the TLDM who should not only optimize his/her objective function but also try to satisfy the SLDM's goal and preference as much as possible.

A. FLDM problem

First, the FLDM solves the following Problem:

$$max\ f_1(x), \quad (5)$$

subject to

$$x \in G.$$

To build membership function, goals and tolerances should be determined first. However, they could hardly be determined without meaningful supporting data.

We should first find the individual best solution f_1^* and individual worst solution $\bar{f}_1$ of (5), where

$$f_1^* = \max_{x \in G} f_1(x), \bar{f}_1 = \min_{x \in G} f_1(x) \quad (6)$$

This data can then be formulated as the following membership function:

$$\mu[f_1(x)] = \begin{cases} 1 & f_1(x) > f_1^*, \\ \frac{f_1(x)-\bar{f}_1}{f_1^*-\bar{f}_1} & \bar{f}_1 \le f_1(x) \le f_1^*, \\ 0 & \bar{f}_1 \ge f_1(x). \end{cases} \quad (7)$$

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Now, we can get the solution of the FLDM problem by solving the following Tchebycheff problem:

$$\max \lambda, \quad (8)$$

subject to

$$x \in G,$$
$$\mu[f_1(x)] \geq \lambda,$$
$$\lambda \in [0,1].$$

The FLDM solution is assumed to be $[\lambda^F, x_1^F, x_2^F, x_3^F, f_1^F]$ where λ^F is satisfactory level.

B. SLDM problem

Second, in the same way, the SLDM independently solves:

$$max\ f_2(x), \quad (9)$$

subject to

$$x \in G.$$

The individual best solution f_2^* and individual worst solution $\bar{f}_2$ of (9), where

$$f_2^* = \max_{x \in G} f_2(x), \bar{f}_2 = \min_{x \in G} f_2(x) \quad (10)$$

This information can then be formulated as the following membership function:

$$\mu[f_2(x)] = \begin{cases} 1 & f_2(x) > f_2^*, \\ \frac{f_2(x) - \bar{f}_2}{f_2^* - \bar{f}_2} & \bar{f}_2 \leq f_2(x) \leq f_2^*, \\ 0 & \bar{f}_2 \geq f_2(x). \end{cases} \quad (11)$$

Now the solution of the SLDM can be obtained by solving the following Tchebycheff problem:

$$max\ \lambda, \quad (12)$$

subject to

$$x \in G,$$
$$\mu[f_2(x)] \geq \lambda,$$
$$\lambda \in [0,1].$$

The FLDM solution is assumed to be $[\lambda^S, x_1^S, x_2^S, x_3^S, f_2^S]$ where λ^S is satisfactory level.

C. *TLDM problem*

Third, the TLDM solves the following problem:

$$max\ f_3(x), \quad (13)$$

subject to

$$x \in G.$$

The individual best solution f_3^* and individual worst solution $\bar{f}_3$ of (13), where

$$f_3^* = \max_{x \in G} f_3(x), \bar{f}_3 = \min_{x \in G} f_3(x) \quad (14)$$

This data can then be formulated as the following membership function:

$$\mu[f_3(x)] = \begin{cases} 1 & f_3(x) > f_3^*, \\ \frac{f_3(x) - \bar{f}_3}{f_3^* - \bar{f}_3} & \bar{f}_3 \leq f_3(x) \leq f_3^*, \\ 0 & \bar{f}_3 \geq f_3(x). \end{cases} \quad (15)$$

Now, we can get the solution of the TLDM problem by solving the following Tchebycheff problem:

$$max\ \lambda, \quad (16)$$

subject to

$$x \in G,$$
$$\mu[f_3(x)] \geq \lambda,$$
$$\lambda \in [0,1].$$

The TLDM solution is assumed to be $[\lambda^T, x_1^T, x_2^T, x_3^T, f_3^T]$ where λ^T is satisfactory level.

Now the solution of the FLDM, SLDM, and TLDM are disclosed. However, three solutions are usually different because of nature between three levels objective functions.

The FLDM knows that using the optimal decisions x_1^F as a control factors for the SLDM, and TLDM are not practical. It is more reasonable to have some tolerance that gives the SLDM, and TLDM an extent feasible region to search for his/her optimal solution. And the SLDM knows that using the optimal decisions x_1^S as a control factors for the TLDM is not practical. It is more reasonable to have some tolerance that gives the TLDM an extent feasible region to search for his/her optimal solution, and reduce searching time or interactions. In this way, the range of decision variable x_1 should be around x_1^F with maximum tolerance t_1 and the following membership function specify $x_1$as:

$$\mu(x_1) = \begin{cases} \frac{x_1 - (x_1^F - t_1)}{t_1} & x_1^F - t_1 \leq x_1 \leq x_1^F, \\ \frac{-x_1 + (x_1^F + t_1)}{t_1} & x_1^F \leq x_1 \leq x_1^F - t_1, \end{cases} \quad (17)$$

And, the range of decision variable x_2 should be around x_2^S with maximum tolerance t_2 and the following membership function specify $x_2$as:

$$\mu(x_2) = \begin{cases} \frac{x_2 - (x_2^S - t_2)}{t_2} & x_2^S - t_2 \leq x_2 \leq x_2^S, \\ \frac{-x_2 + (x_2^S + t_2)}{t_2} & x_2^S \leq x_2 \leq x_2^S - t_2, \end{cases} \quad (18)$$

The FLDM goals may reasonably consider $f_1 \geq f_1^F$ is absolutely acceptable and $f_1 \leq f_1^F$ is absolutely unacceptable, and that the preference with $[f_1^*, f_1^F]$ is linearly

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increasing. This due to the fact that the SLDM obtained the optimum at (x_1^S, x_2^S, x_3^S), which in turn provides the FLDM the objective function values $\hat{f}_1$, make any $f_1 \geq \hat{f}_1 = f_1(x_1^S, x_2^S, x_3^S)$ unattractive in practice.

The following membership functions of the FLDM can be stated as:

$$\dot{\mu}[f_1(x)] = \begin{cases} 1 & f_1(x) > f_1^F, \\ \frac{f_1(x) - \hat{f}_1}{f_1^F - \hat{f}_1} & \hat{f}_1 \leq f_1(x) \leq f_1^F, \\ 0 & \hat{f}_1 \geq f_1(x). \end{cases} \quad (19)$$

Second, the SLDM goals may reasonably consider the $f_2 \geq f_2^S$ is absolutely acceptable and $f_2 \leq f_2^S$ is absolutely unacceptable, and that the preference with $[\hat{f}_2, f_2^S]$ is linearly increasing. In this way, the SLDM has the following membership functions for his/her goal:

$$\dot{\mu}[f_2(x)] = \begin{cases} 1 & f_2(x) > f_2^S, \\ \frac{f_2(x) - \hat{f}_2}{f_2^S - \hat{f}_2} & \hat{f}_2 \leq f_2(x) \leq f_2^S, \\ 0 & \hat{f}_2 \geq f_2(x). \end{cases} \quad (20)$$

Third, the TLDM goals may reasonably consider the $f_3 \geq f_3^T$ is absolutely acceptable and $f_3 \leq f_3^T$ is absolutely unacceptable, and that the preference with $[\hat{f}_3, f_3^T]$ is linearly increasing. In this way, the TLDM has the following membership functions for his/her goal:

$$\dot{\mu}[f_3(x)] = \begin{cases} 1 & f_3(x) > f_3^T, \\ \frac{f_3(x) - \hat{f}_3}{f_3^S - \hat{f}_3} & \hat{f}_3 \leq f_3(x) \leq f_3^T, \\ 0 & \hat{f}_3 \geq f_3(x). \end{cases} \quad (21)$$

Finally, in order to generate the satisfactory solution, which is also a Pareto optimal solution with overall satisfaction for all decision-makers, we can solve the following Tchebycheff problem.

$$max\, \beta, \quad (22)$$

subject to

$$\dot{\mu}[f_1(x)] \geq \beta,$$
$$\mu(x_1) \geq \beta,$$
$$\mu(x_2) \geq \beta,$$
$$x \in G,$$
$$t_i > 0,$$
$$\beta \in [0,1].$$
$$\dot{\mu}[f_2(x)] \geq \beta,$$
$$\dot{\mu}[f_3(x)] \geq \beta,$$

V. Numerical Example

[1st level]

$$\max_{x_1} f_1(x) = 2x_1^2 + x_2 + x_3 + 3x_1,$$

Where, x_2, x_3 solves

[2nd level]

$$\max_{x_2} f_2(x) = -x_1 + 5x_2^2 + 2x_3,$$

Where, x_3 solves

[3rd level]

$$\max_{x_3} f_3(x) = x_1 - x_2 + 3x_3^2,$$

subject to:

$$\xi_1 x_1 \mid \xi_2 x_2 \mid \xi_3 x_3 \leq 7,$$
$$\xi_4 x_1 + \xi_5 x_2 + \xi_6 x_3 \leq 5,$$
$$x_1, x_2, x_3 \geq 0.$$

Assume that the rough parameters are defines as:

$$\xi_1 \sim N(\rho_1, 1)_{\text{with}}\, \rho_1 = ([1,2],[0,3])$$
$$\xi_2 \sim N(\rho_2, 2)_{\text{with}}\, \rho_2 = ([3,4],[2,5])$$
$$\xi_3 \sim N(\rho_3, 1)_{\text{with}}\, \rho_3 = ([2,3],[1,5])$$
$$\xi_4 \sim N(\rho_4, 2)_{\text{with}}\, \rho_4 = ([2,3],[1,5])$$
$$\xi_5 \sim N(\rho_5, 1)_{\text{with}}\, \rho_5 = ([2,4],[1,4])$$
$$\xi_6 \sim N(\rho_6, 2)_{\text{with}} \rho_6 = ([2,3],[1,3])$$

We set $\theta_r = \eta_r = 0.9$

Now by using theorem 2, the equivalent crisp problem which equivalent to three-level quadratic programming problem under rough parameters in constraints is:

[1st level]

$$\max_{x_1} f_1(x) = 2x_1^2 + x_2 + x_3 + 3x_1,$$

Where, x_2, x_3 solves

[2nd level]

$$\max_{x_2} f_2(x) = -x_1 + 5x_2^2 + 2x_3,$$

Where, x_3 solves

[3rd level]

$$\max_{x_3} f_3(x) = x_1 - x_2 + 3x_3^2,$$

subject to:

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$$2.4x_1 + 4.4x_2 + 4.2x_3 \leq 7,$$
$$4.2x_1 + 3.4x_2 + 2.6x_3 \leq 5,$$
$$x_1, x_2, x_3 \geq 0.$$

Then, calculating Trust for every rough coefficient using trust measure function in theorem 2.

$Tr(\xi_1) = 0.9$, $Tr(\xi_2) = 0.9$,
$Tr(\xi_3) = 0.9$, $Tr(\xi_4) = 0.9$,
$Tr(\xi_5) = 0.75$, $Tr(\xi_6) = 0.7$

So ω = 0.7

To solve the equivalent crisp problem of three-level quadratic programming problem under rough parameters in constraints:

First: the FLDM solves his\her problem as following:

Find individual optimal solution by solving (5), we get:

$(f_1^*, \overline{f_1}) = (6.405896, 0)$

By using (7), the FLDM build membership function $\mu f_1(x)$ then solve the Tchebycheff problem (8) as follows:

$$\max \lambda,$$

subject to

$$2x_1^2 + x_2 + x_3 + 3x_1 - 6.405896\lambda \leq 0,$$
$$x \in G,$$
$$\lambda \in [0,1].$$

Whose solution is

$\lambda^F = 0.5$, $(x_1^F, x_2^F, x_3^F) = (0.7194063, 0, 0.760959)$, $f_1^\beta = 0.5$

Second, the SLDM solves his / her problem as follows: Finds individual optimal solutions by solving (9), we get:

$(f_2^*, \overline{f_2}) = (10.81315, -1.19)$.

By using (11), the SLDM build membership function $\mu f_2(x)$ then solve the Tchebycheff problem (12) as follows:

$$\max \lambda.$$

subject to

$$-x_1 + 5x_2^2 + 2x_3 - 12.00315\lambda \leq -1.19,$$
$$x \in G,$$
$$\lambda \in [0,1].$$

Whose solution is

$\lambda^S = 0.43$, $(x_1^S, x_2^S, x_3^S) = (0.64749, 0.726, 0.069)$, $f_2^S = 3.72509$.

Third, the TLDM solves his / her problem as follows:

Finds individual optimal solutions by solving (13), we get:

$(f_3^*, \overline{f_3}) = (8.333333, -0.186)$

By using (15), the TLDM build membership function $\mu f_3(x)$ then solve the Tchebycheff (16) problem as follows:

$$\max \lambda,$$

subject to.

$$x_1 - x_2 + 3x_3^2 - 8.519\lambda \leq -0.186,$$
$$x \in G,$$
$$\lambda \in [0,1].$$

Whose solution is

$\lambda^T = 0.39$, $(x_1^T, x_2^T, x_3^T) = (0.165, 0.3222, 1.235)$,
$f_3^T = 4.415522$.

Finally, calculating the membership function for Tchebycheff problem:

We assume the FLDM'S control decision ($x_1^F = 0.7194063$) with the tolerance 0.1798, and assume the SLDM'S control decision ($x_1^S = 0.726$) with the tolerance 0.1815.

By using (25) – (27) calculating membership functions $\tilde{\mu}$, then solves the Tchebycheff problem (22) as follows:

$$\max \delta,$$

subject to

$$x \in G,$$
$$2x_1^2 + x_2 + x_3 + 3x_1 + 1.17\delta \geq 4.373,$$
$$-x_1 + 5x_2^2 + 2x_3 - 0.9036\delta \geq 2.823,$$
$$x_1 - x_2 + 3x_3^2 + 1.425\delta \geq 3.638,$$
$$x_1 - 0.15\delta \geq 0.3755,$$
$$x_1 + 0.15\delta \geq 0.6755,$$
$$x_2 - 0.2\delta \geq 0.526,$$
$$x_2 + 0.2\delta \geq 0.926,$$
$$\delta \in [0,1].$$

Whose, optimal solution is:

$\delta^0 = 0.5$, $(x_1^0, x_2^0, x_3^0) = (0.8797, 0.746, 0.493)$
$(f_1^0, f_2^0, f_3^0) = (5.425844, 2.888, 2.3548)$ Overall satisfaction for both decisions – makers

VI. Conclusion

This paper presented a three-level quadratic programming problem with random rough coefficient in constraints. At the first phase of the solution algorithm and to avoid the complexity of this problem, we began by converting the rough nature of this problem into equivalent crisp problem. At the second phase, a membership function was constructed to develop a fuzzy model for obtaining the optimal solution of the three-level quadratic programming problem.

However, there are many other aspects, which should by explored and studied in the area of a multi-level optimization such as:

1. Large scale multi-level multi-objective non-linear programming problem with rough parameters in objective.
2. Large scale multi-level multi-objective non-linear programming problem with rough parameters in constraints.
3. Large scale multi-level multi-objective non-linear programming problem with rough parameters in both objectives and constraints.

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